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The movement of water through the landscape can be investigated at different scales. This study dealt with the interrelation between bedrock lithology and the geometry of the overlying drainage systems. Parameters of fractal analysis, such as fractal dimension and lacunarity, were used to measure and quantify this relationship.

# Lithologic Control on the Scaling Properties of the First-Order Streams of Drainage Networks: A Monofractal Analysis

The interrelation between bedrock lithology and the geometry of the drainage systems has been widely studied in the last decades. The quantification of this linkage has not yet been clearly established. Several studies have selected river basins or regularly shaped areas as study units, assuming them to be lithologically homogeneous. This study considered irregular distributions of rock types, establishing areas of the soil map (1:25,000) with the same lithologic information as study units. The tectonic stability and the low climatic variability of the study region allowed effective investigation of the lithologic controls on the drainage networks developed on the plutonic rocks, the metamorphic rocks, and the sedimentary materials existing in the study area. To exclude the effect of multiple in- and outflows in the lithologically homogeneous units, we focused this study on the first-order streams of the drainage networks. The geometry of the hydrologic features was quantified through traditional metrics of fluvial geomorphology and scaling parameters of fractal analysis, such as the fractal dimension, the reference density, and the lacunarity. The results demonstrate the scale invariance of both the drainage networks and the set of first-order streams at the study scale and a relationship between scaling in the lithology and the drainage network.

**Drainage networks** have been studied in hydrology (De Bartolo et al., 2000), geology (Howard, 1967), and geomorphology (Montgomery and Dietrich, 1992). Each of these disciplines focuses on different drainage network properties and uses different study scales. That complicates reconciling and using the collected hydrologic and geologic data in a consistent manner (Soulsby et al., 2008; Lin, 2003).

Geologists and geomorphologists have noted close relationships between bedrock lithology and the structure of the overlying drainage networks (Carlston, 1963). Visual analysis of drainage networks and the identification of patterns in these networks have been basic tools commonly used in photogeology to discriminate materials according to their lithology, which is an essential step in the development of geologic and geomorphological mapping at different scales (Soil Survey Division Staff, 1966). More recently, the influence of bedrock lithology on the configuration of flow systems has been reaffirmed (Gaudio et al., 2006; Bloomfield et al., 2011). Metrics to quantify this relationship are still being developed.

Drainage networks have complex shapes that require appropriate geometric measures to be quantified. The hierarchical classification of streams according to the branching structure of the drainage networks and the quantification of physical properties such as the length of the streams in the first half of the previous century demonstrated scaling properties of drainage networks (Horton, 1932, 1945; Strahler, 1957). After the introduction of fractal geometry in the natural sciences (Mandelbrot, 1983), many researchers have investigated and confirmed the fractal and multifractal behavior of drainage networks (Rodríguez-Iturbe and Rinaldo, 1997; De Bartolo et al., 2000, 2004, 2006). Recent studies of the lithologic controls on the fractal and multifractal parameters of river networks (Gaudio et al., 2006; Dombradi et al., 2007) have analyzed areas, ranging from 100 to 10,000 km<sup>2</sup>, considered as lithologically homogeneous. In other words, the differences in scaling properties of drainage networks are interpreted using a single dominant lithology in each of those networks.

Drainage networks are abundant that have developed in areas of heterogeneous lithology. The need and feasibility of analyzing the lithologic control on hydrologic features in areas based on lithologic mapping have been recently suggested by Bloomfield et al. (2011).

The objective of this work was to evaluate the scaling properties of drainage networks within lithologic units and to research the relationships between scaling parameters of their geometric shapes. To exclude the effect of multiple in- and outflows in a lithologic unit, we focused this study on the first-order streams of the drainage networks.

## Study Area Overview

The study area (1001 km<sup>2</sup>) is located in the western half of the Iberian Peninsula (Fig. 1), between the Spanish provinces of Zamora and Salamanca, and it corresponds to the specific wine production area of the Protected Designation of Origin “Arribes”.

The dendritic drainage networks of the study area belong to the Douro River basin and specifically to the left-side tributary system of the Douro River. The study area is bounded to the north and west by the Douro River, to the southwest by the Agueda River, and it is crossed east–southeast to west–northwest by other main rivers such as the Tormes, Huebra, and Uces rivers, sorted by decreasing magnitude.

The geology of the region was formed by the main orogenies that have sculpted the current configuration of the Iberian Peninsula. The plutonic rocks present in the area emerged during the

Hercynian orogeny in the late Paleozoic era. The metamorphism caused by this movement transformed the oldest materials deposited during the Cambrian. Later, during the Mesozoic and the beginning of the Cenozoic, the area was part of an inland sea. During this period, the plutonic rocks (mainly granite) and the metamorphic rocks (ortogneisses, paragneisses, pelitic and psammitic metasediments, and others) were covered by sediments. Then, the Alpine orogeny balanced the Iberian Peninsula to the west and the inland sea was drained to the Atlantic Ocean through the cracks of the Hercynian basement. This process eroded the tertiary sediments, which remain only in a few places of the highest lands of the region.

From a geomorphological point of view, the study area is divided into two well-defined units, the peneplain, which includes the higher lands of the region, and the deep valleys of the rivers that go across it. The sharp contrast between the pronounced steepness of the valleys and the peneplain determines the climatic variations of the region.

According to the Köppen–Geiger climate classification (Köppen, 1918; Köppen and Geiger, 1928), the climatic conditions of the valleys are classified as Csa (mild humid climate with a wet winter and a dry and hot summer), while the climatic conditions of the peneplain are classified as BSk (cold semiarid climate characterized by grasslands where the mean annual precipitation varies according to the mean annual temperature). The main difference between these two classes is the temperature regime, due to the protection of the valleys from the dominant winds coming from the Atlantic Ocean (Gómez-Miguel et al., 2011).

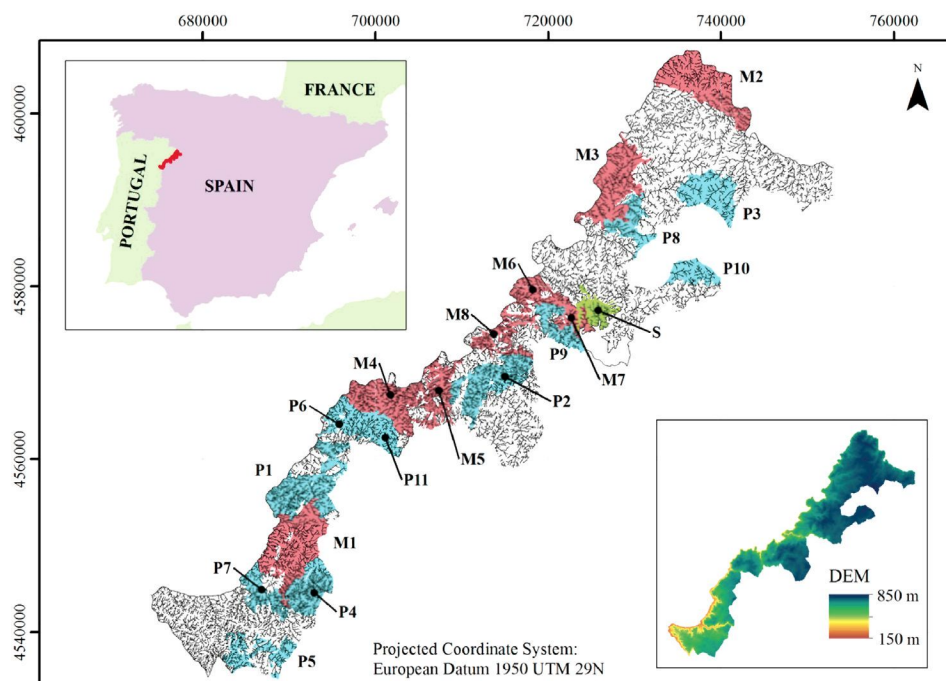


Fig. 1. Overview of the study area showing the lithologic units studied, with an inset of a digital elevation model (DEM) of the study area.



The tectonic stability and the low climatic variability of the region are essential preconditions for the effective study of the lithologic controls on the drainage network (Dombradi et al., 2007; Bloomfield et al., 2011).

The land use is also noteworthy. According to the National Geographic Information System of Agricultural Fields (SIGPAC), the area of the average field in the studied zone is only 0.74 ha, and usually the borders between fields are stone walls built some centuries ago. These walls represent an efficient soil conservation practice, but they also condition the natural process of channel incision and hence the development of the drainage network.

## Materials and Methods

### Data Set

This study was performed using two sources of information: the lithologic information of the soil map of the region, scale 1:25,000 (Gómez-Miguel et al., 2011) and the drainage network, digitized from aerial photographs.

The lithologic information of the soil map shows that there are 257 lithologic units, 176 of which belong to three source rock types: plutonic rocks (P), metamorphic rocks (M), and sedimentary materials (S). To carry out this study, 20 lithologic units with areas ranging from 10 to 50 km<sup>2</sup> were selected. Eleven of the 20 selected units correspond to the plutonic rocks present in the area and are called P1 to P11 from the largest to the smallest; eight units represent the metamorphic rocks (M1–M8); and only one is representative of the sedimentary materials (S).

The drainage network was obtained through photointerpretation by stereoscopic viewing of 267 aerial photographs, 23 by 23 cm, scale 1:18,000. We considered all evidence on the terrain of continuous or intermittent watercourses, and we have considered as first-order streams those streams of the drainage system that do not have tributaries. Once identified on the photographs, the information about the drainage network was translated by hand to a vector file using the available orthophotographs of the region with a resolution of 25 cm as spatial reference.

### Box-Counting Method

The method used for the estimation of the fractal dimension in this work was the box-counting method, which belongs to the family of fixed-size algorithms. This method represents one of the simplest methods of scaling analysis from a computational point of view and it has been widely used to obtain

generalized fixed-size algorithms successfully applied to the multifractal characterization of river networks (De Bartolo et al., 2000, 2004, 2006).

According to Rodríguez-Iturbe and Rinaldo (1997), to obtain the box-counting dimension the studied geometric object has to be covered with a regular grid of side  $r$ . Then the number  $N(r)$  of grid boxes that contain some part of the object is counted. The  $r$  value is progressively reduced to obtain a series of smaller sizes and corresponding  $N(r)$  numbers. As  $r \rightarrow 0$ ,  $\log N_i / \log(1/s_i)$  converges to a finite value defined as the box-counting dimension. In this expression,  $N_i = N(r_i)$  and  $s_i = r_i/r_1$ , where  $r_1$  is the first grid size considered for the box-counting dimension estimation. Figures 2 and 3 illustrate the procedure. The box-counting dimension corresponds to the slope gradient of the regression line shown in Fig. 2.

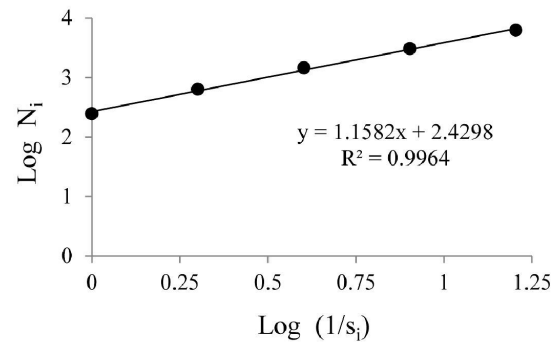


Fig. 2. Log-log plot of the number of boxes that intersect the studied feature ( $N_i$ ) vs. the size of the intersected boxes ( $s_i$ ) for the drainage system of the lithologic unit P5.

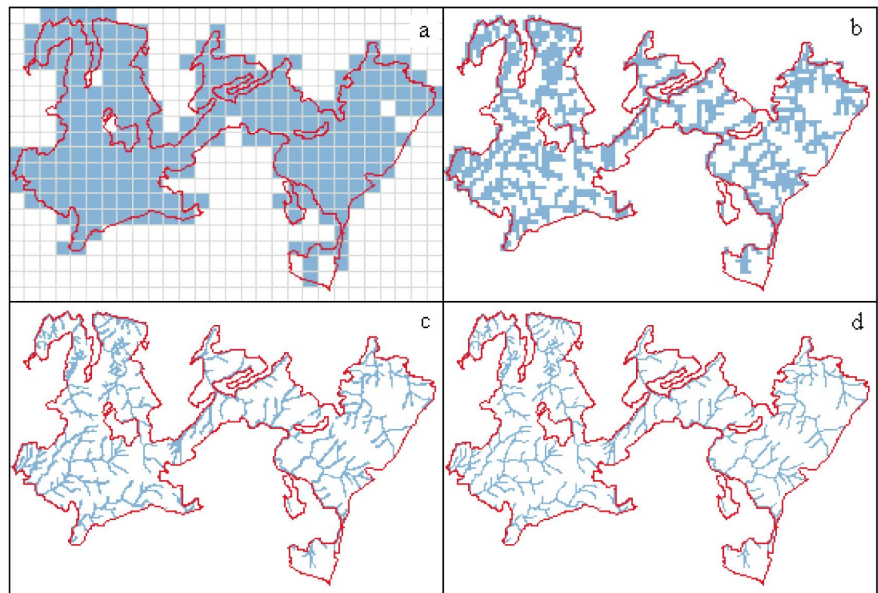


Fig. 3. Box-counting method applied to the drainage network of the lithologic unit P5: (a) boxes of 320 by 320 m occupied by the drainage network ( $s_1 = 320/320 = 1$ ); (b) boxes of 80 by 80 m ( $s_2 = 80/320 = 1/4$ ); (c) boxes of 20 by 20 m ( $s_3 = 20/320 = 1/16$ ); and (d) the original drainage network.

In this study case, the whole drainage network was totally covered by a square of 81,920 by 81,920 m centered on the central point of the extension of the layer (X\_UTM:713583; Y\_UTM:4570899; coordinate system: European Datum 1950 29N). This square was divided into four similar boxes of 40,960 by 40,960 m. This step was repeated until the initial square had been divided into 16,777,216 boxes of 20 by 20 m. Due to the irregular shape of the lithologic units and the way in which the boxes of different sizes represent the shape of the lithologic unit (Fig. 3a), the box sizes considered as representative for the fractal dimension estimation were 320, 160, 80, 40, and 20 m. According to the above nomenclature,  $r_1 = 320$  m,  $r_2 = 160$  m,  $r_3 = 80$  m,  $r_4 = 40$  m, and  $r_5 = 20$  m (Fig. 3).

The method used to estimate lacunarity was adapted from the gliding box method described by Borys (2009). Lacunarity was obtained by considering the number of empty boxes (20 by 20 m) within each box of 320 by 320 m.

All confidence intervals shown in this work are at the 95% confidence level.

## Results

### Quantitative Framework of Fluvial Geomorphology

Eight variables describing the area, the drainage system, and the first-order streams of the 20 lithologic units are provided in

Table 1. The drainage density values refer to the whole lithologic unit. According to these values, there are no differences between plutonic and metamorphic units; the drainage density for the plutonic rocks is  $4.36 \pm 0.50$  km/km<sup>2</sup> while the value for the metamorphic rocks is  $4.22 \pm 0.43$  km/km<sup>2</sup>. Similarly, the average lengths of the first-order streams are almost equal for the plutonic units ( $239.9 \pm 27.2$  m) and the metamorphic units ( $235.7 \pm 27.9$  m). In both cases, for the drainage density and the average length of the first-order streams, the values for the sedimentary unit are lower. However, as there is only one sedimentary unit, we avoid any statistical comparisons because we have no evidence of its representativeness. The presence of main rivers bounding or crossing the lithologic unit in all metamorphic units suggests some preference or easiness of water erosion of this kind of material compared with the plutonic rocks.

### Fractal Dimension of the Drainage Systems

According to the box-counting values reported in Table 2, the fractal dimension of the drainage systems,  $D$ , of the plutonic units is  $1.139 \pm 0.027$ , while the  $D$  of the metamorphic units is  $1.108 \pm 0.016$ . The value of  $D$  for the sedimentary unit is  $<1$ . The  $a$  value reported in Tables 2 and 3 represents the intersection of the regression line with the ordinate axis (Fig. 2). The high values of the coefficient of determination ( $R^2$ ),  $>0.9920$  in all the estimations, suggest scale invariance of the studied features at the study scale. It is worth noting that, for example, for the lithologic unit P5, the  $R^2$  of the fit with box sizes of 5120, 2560, 1280, 640, and 320 m is higher (0.9996) than the  $R^2$  of the fit

Table 1. Quantitative description of the 20 lithologic units and their drainage systems.

Unit	Drainage system					First-order streams		
	Area	Length	Drainage density	Highest order	Tributary to a main river	Length	No. of streams	Avg. length
	km <sup>2</sup>	m	km/km <sup>2</sup>			m		m
P1	38.13	176,108	4.62	fourth	yes	101,974	471	216.5
P2	32.98	170,772	5.18	sixth	no	104,417	495	210.9
P3	25.63	76,424	2.98	fourth	no	47,006	147	319.8
P4	22.76	123,104	5.41	fifth	no	76,559	341	224.5
P5	18.56	100,383	5.41	fourth	no	55,024	267	206.1
P6	18.29	76,937	4.21	fourth	yes	46,247	212	218.1
P7	15.17	74,959	4.94	sixth	yes	42,466	193	220.0
P8	15.13	53,631	3.54	fifth	no	31,213	116	269.1
P9	13.51	58,502	4.33	fourth	yes	34,993	165	212.1
P10	12.80	38,251	2.99	third	no	22,822	67	340.6
P11	10.42	45,642	4.38	fourth	yes	26,542	132	201.1
M1	44.39	196,648	4.43	fifth	yes	107,620	449	239.7
M2	42.06	144,943	3.45	fifth	yes	90,422	302	299.4
M3	37.59	123,731	3.29	sixth	yes	71,082	242	293.7
M4	34.27	145,950	4.26	fourth	yes	91,522	383	239.0
M5	16.94	84,013	4.96	fourth	yes	47,752	249	191.8
M6	15.19	61,954	4.08	fifth	yes	35,494	172	206.4
M7	13.28	54,258	4.09	fourth	yes	33,669	145	232.2
M8	10.13	52,885	5.22	sixth	yes	31,961	174	183.7
S	10.77	26,939	2.50	fourth	no	17,935	99	181.2



(0.9964) with the box sizes considered in the study (320, 160, 80, 40, and 20 m). Although the  $R^2$  is higher with those pixel sizes, the scale invariance suggested by this value of  $R^2$  is attributable to the shape of the lithologic unit and not to the drainage system because the smallest size is 320 m and, according to Fig. 3, this box size does not reflect drainage information.

## Fractal Dimension of the First-Order Streams

Results in Table 3 show that the fractal dimension of the first-order streams,  $D_1$ , is slightly higher in the plutonic rock units ( $1.003 \pm 0.022$ ) than in the metamorphic rock units ( $0.976 \pm 0.015$ ). Here again the values are below or very close to 1, but the  $R^2$  values suggest clear scaling properties in all the sets of first-order streams.

Table 2. Box-counting results including fractal dimension ( $D$ ) and reference density ( $a$ ) for the drainage systems of the 20 lithologic units.

Unit	$D$	$a$	$R^2$	No. of grid boxes occupied by the drainage system with different box sizes											
				40,960 m	20,480 m	10,240 m	5120 m	2560 m	1280 m	640 m	320 m	160 m	80 m	40 m	20 m
P1	1.1671	2.6746	0.9938	1	2	3	7	18	44	132	418	1163	2640	5427	11,051
P2	1.1979	2.6269	0.9926	4	4	4	5	13	34	115	369	1077	2484	5241	10,627
P3	1.0710	2.4057	0.9986	1	2	2	4	9	26	83	241	559	1172	2359	4802
P4	1.2143	2.4685	0.9921	1	2	2	4	11	23	79	255	756	1786	3760	7691
P5	1.1582	2.4298	0.9964	1	2	2	4	11	29	84	246	636	1456	3048	6221
P6	1.1415	2.3446	0.9943	1	1	1	3	9	22	62	197	533	1180	2400	4851
P7	1.1700	2.2990	0.9933	1	1	1	3	7	22	58	175	495	1113	2305	4678
P8	1.0831	2.2374	0.9984	1	1	1	4	9	23	58	163	385	802	1663	3347
P9	1.1302	2.2336	0.9957	1	1	2	4	7	19	52	155	405	885	1822	3672
P10	1.0755	2.1008	0.9988	1	2	3	5	7	15	44	120	277	579	1191	2406
P11	1.1218	2.1322	0.9952	1	1	2	3	6	16	40	122	321	696	1410	2841
M1	1.1474	2.7328	0.9942	1	4	4	7	16	48	147	480	1319	2892	5942	12,060
M2	1.0969	2.6642	0.9968	1	2	3	6	15	42	127	424	1059	2245	4544	9163
M3	1.0962	2.5951	0.9976	1	2	3	7	15	41	124	366	892	1899	3883	7835
M4	1.1349	2.6296	0.9942	1	1	2	6	15	41	120	379	1030	2238	4572	9188
M5	1.1213	2.4026	0.9952	2	2	3	7	11	28	71	228	592	1309	2627	5274
M6	1.0762	2.3209	0.9962	1	1	2	5	10	25	65	191	482	981	1960	3947
M7	1.0835	2.2538	0.9980	1	1	2	5	10	21	60	168	400	845	1731	3452
M8	1.1038	2.2236	0.9957	2	2	2	2	6	20	54	152	386	840	1673	3348
S	0.9983	2.0397	0.9972	1	1	2	4	8	18	42	102	236	455	857	1703

Table 3. Box-counting results including fractal dimension ( $D_1$ ) and reference density ( $a_1$ ) for the first-order streams of the drainage systems of the 20 lithologic units.

Unit	$D_1$	$a_1$	$R^2$	No. of grid boxes occupied by the drainage system with different box sizes											
				40,960 m	20,480 m	10,240 m	5120 m	2560 m	1280 m	640 m	320 m	160 m	80 m	40 m	20 m
P1	1.0195	2.6249	0.9948	1	2	3	7	18	44	125	381	935	1859	3494	6749
P2	1.0558	2.6014	0.9927	3	3	3	4	12	33	113	353	921	1888	3599	6933
P3	0.9566	2.3353	0.9996	1	2	2	4	9	25	81	211	433	824	1563	3058
P4	1.0780	2.4390	0.9920	1	2	2	4	11	23	77	241	646	1352	2595	5042
P5	1.0133	2.3642	0.9971	1	1	2	4	11	28	79	215	494	1002	1905	3669
P6	0.9989	2.3032	0.9968	1	1	1	3	9	22	62	186	432	843	1595	3086
P7	1.0189	2.2401	0.9948	1	1	1	3	7	20	54	157	387	760	1445	2776
P8	0.9586	2.1624	0.9989	1	1	1	4	8	22	54	139	296	561	1060	2036
P9	0.9959	2.1818	0.9965	1	1	2	4	7	18	49	140	328	633	1204	2305
P10	0.9610	2.0199	0.9995	1	2	3	5	7	15	40	102	212	395	767	1499
P11	0.9814	2.0772	0.9955	1	1	2	3	6	15	38	109	259	488	912	1743
M1	0.9892	2.6736	0.9969	1	4	4	7	16	48	144	437	1012	1931	3672	7071
M2	0.9818	2.5994	0.9987	1	2	3	6	15	41	124	379	826	1587	3043	5933
M3	0.9559	2.5217	0.9995	1	2	3	6	14	39	120	324	670	1248	2410	4692
M4	1.0191	2.5724	0.9955	1	1	2	6	15	41	115	340	827	1625	3105	6000
M5	0.9627	2.3605	0.9960	2	2	3	7	11	28	69	211	484	919	1678	3187
M6	0.9470	2.2376	0.9986	1	1	2	5	9	22	60	165	354	648	1220	2367
M7	0.9819	2.1806	0.9986	1	1	2	5	10	21	58	144	317	600	1171	2252
M8	0.9668	2.1844	0.9969	2	2	2	2	5	19	52	142	320	615	1127	2158
S	0.8901	2.0037	0.9978	1	1	2	4	8	18	41	96	200	351	621	1191

Plotting the values of  $D_1$  vs.  $D$ , we find two different results depending on the bedrock lithology (Fig. 4). The result obtained for the eight metamorphic rock units does not show any trend, while the plot of the plutonic rock units reflects a positive linear relationship. This clear trend suggests some regularity in the eroding process of the granitic rocks, probably due to their internal configuration. This regularity does not appear in the metamorphic units, whose materials are affected by the directions of the metamorphic process.

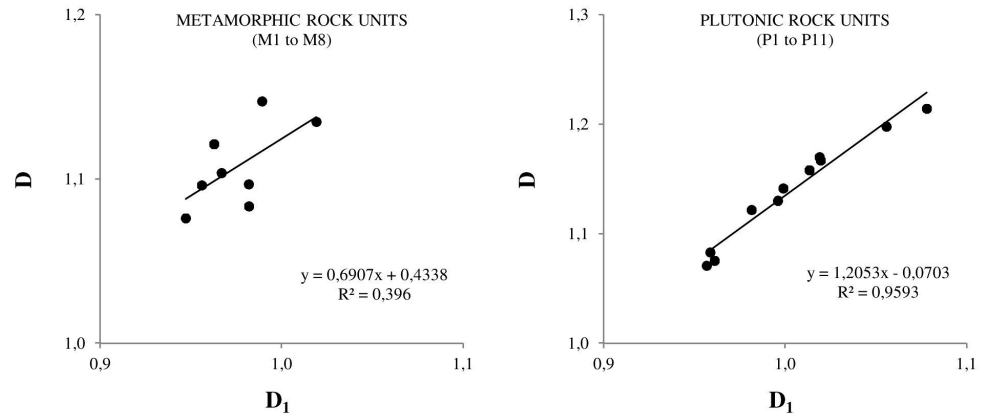


Fig. 4. Plots of the fractal dimension of the first-order streams ( $D_1$ ) vs. the fractal dimension of the drainage system ( $D$ ).

## Intercepts

The parameters  $a$  for the drainage systems and  $a_1$  for the first-order streams shown in Tables 2 and 3 have been typically related to the reference density or the number of the counted boxes of the largest size. The very strong correlation between parameters  $a$  and  $a_1$  in the two studied scaling laws reflects a similarity in scaling of both the drainage networks and the set of their first-order streams (Fig. 5). This relationship seems not to be affected by lithology and probably reflects the effects of other environmental factors.

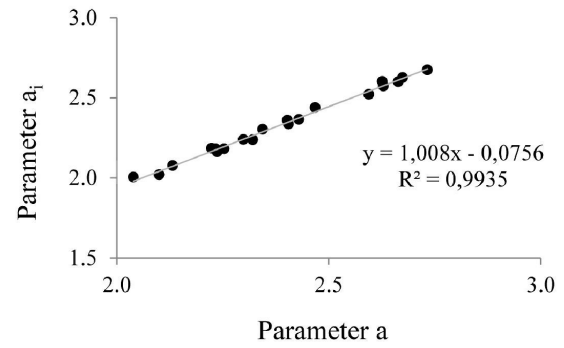


Fig. 5. Plots of the reference density parameter of the drainage system ( $a$ ) vs. the reference density parameter of the first-order streams ( $a_1$ ).

Plotting the values resulting from the box-counting method for the two studied scaling laws shows a weak dependence of parameter  $a$  on the fractal dimension  $D$  (Fig. 6). This supports the assumption that the fractal dimension and the reference density of the two features are controlled by different environmental factors. However, the lithologic units with similar source rock type, which are represented in the same color, appear to be grouped (Fig. 6).

## Lacunarity

Lacunarity was calculated for both the drainage network of each lithologic unit ( $\Lambda$ ) and the set of its first-order streams ( $\Lambda_1$ ). The  $\Lambda$  of the plutonic units is  $1.00465 \pm 0.00065$ , while the  $\Lambda$  of the metamorphic units is  $1.00408 \pm 0.00025$ . In the case of  $\Lambda_1$ , the value for the plutonic rock units is  $1.00203 \pm 0.00023$  and the value for the metamorphic rock units is  $1.00189 \pm 0.00014$ . Like the values of  $D$  and  $D_1$ , the values of  $\Lambda$  and  $\Lambda_1$  for the sedimentary unit are the lowest of the studied units (1.00230 and 1.00121, respectively). The linear relationship between  $\Lambda_1$  and  $\Lambda$ , with  $R^2 = 0.8337$ , suggests a similarity in the two scaling laws (Fig. 7).

The results reflect that  $\Lambda$  and  $\Lambda_1$  increase as  $D$  and  $D_1$  increase (Fig. 8). This finding supports the idea that the fractal dimension and the lacunarity are controlled by the same environmental factors. The linear relationship between  $\Lambda$  and  $D$  ( $R^2 = 0.7765$ ) is stronger than the relationship between  $\Lambda_1$  and  $D_1$  ( $R^2 = 0.6331$ ).

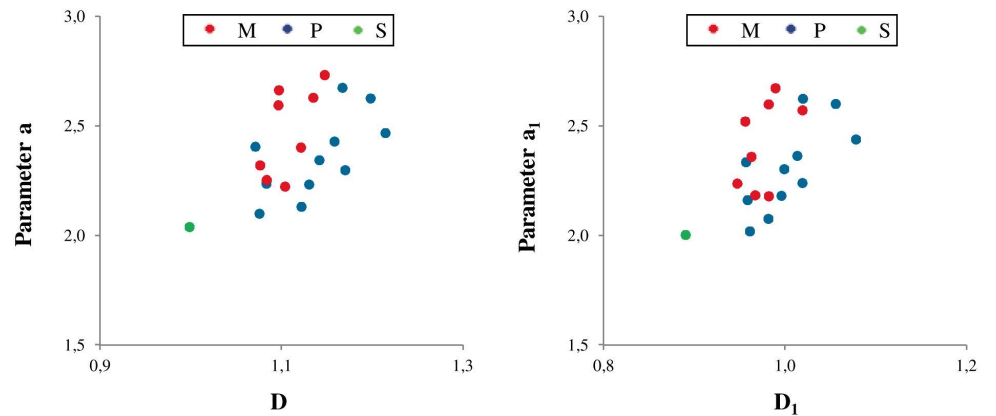


Fig. 6. Plots of the fractal dimension ( $D$  and  $D_1$ ) vs. the reference density parameter ( $a$  and  $a_1$ ) for the two studied scaling laws, drainage system on the left and first-order streams on the right, for the plutonic (P), metamorphic (M), and sedimentary (S) lithologic units.



As in Fig. 6, the lithologic units with similar rock type are shown grouped in Fig. 8.

## Discussion

### Suitability of the Assumed Scaling Model

The method and the scale range used for the estimation of the fractal dimension seem to be appropriate, resulting in  $R^2$  values that suggest the scale invariance of the studied hydrologic features at the study scale.

The box-counting method is known to be very sensitive to border effects, and it is not able to determine the correct dimension for the negative moment order of the multifractal spectrum (De Bartolo et al., 2000; Gaudio et al., 2004). However, the procedure followed here, with an initial box size (320 m) significantly smaller than the study unit, suggests a suitable sensitivity to the study objective. If larger box sizes were used, we would estimate the scaling properties of the shape of the lithologic unit and not of the drainage network.

The manual editing of the drainage network could introduce some identification and digitization errors, but using this data set we are free of other methodological errors related to the channel network extraction from digital elevation models (Helmlinger et al., 1993).

### Properties of Fluvial Geomorphology

The values obtained for the plutonic and the metamorphic units are very similar. These results support the fact that in Gaudio et al. (2006), the metamorphic and plutonic rocks were grouped as the same source rock type. The reported values of drainage density for the plutonic and metamorphic rocks are lower than those presented by Gaudio et al. (2006) for the plutonic and metamorphic areas of the Calabrian region (4.93–6.24 km/km<sup>2</sup>). These differences could be based on the topographic differences between this part of Italy and our study area of Spain. Also, it is possible that the stone walls present in the region, which divide the landscape everywhere, play a role as a soil conservation practice that affects the drainage density.

### Fractal Scaling Parameters

The presented values of the fractal dimension of the drainage networks are clearly lower than those previously reported by Gaudio et al. (2006) and Dombradi et al. (2007) for areas having similar source rock types to the lithologic units studied here. However, the values of  $D$  for the drainage networks are close to 1.1, which together

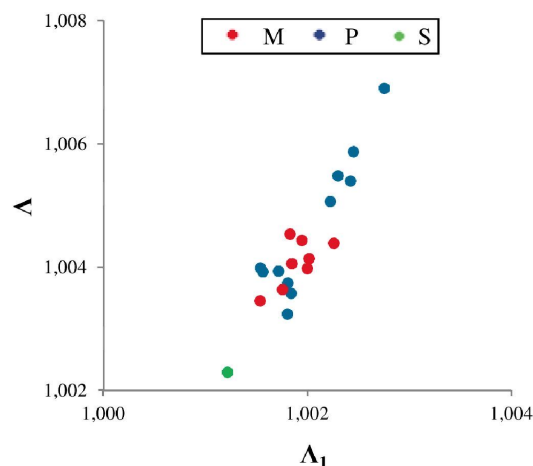


Fig. 7. Plot of the lacunarity of the drainage system ( $\Lambda$ ) vs. the lacunarity of the first-order streams ( $\Lambda_1$ ) for the plutonic (P), metamorphic (M), and sedimentary (S) lithologic units.

with 1.5 and 1.7 are the “more likely” fractal dimension values of river networks described by Claps and Oliveto (1996).

The case of the values of  $D_1$  is significant in that almost all the lithologic units present values  $<1$ . The interpretation of the results of  $D$  and  $D_1$  is linked to the range of box sizes used in the estimation of the fractal dimension. Values of  $D_1$  ranging from 0 to 1 represent intermediate situations between the existence of many streams shorter than 20 m and separated  $>320$  m ( $D_1 = 0$ ) and the presence of several first-order streams crossing the lithologic unit from side to side and separated from each other by  $>320$  m ( $D_1 = 1$ ). Values of  $D_1 >1$  are related to the highest drainage densities. The only lithologic unit (S) with a  $D$  value  $<1$  shows many scattered in- and outflows resulting in a drainage system without a clear branching structure.

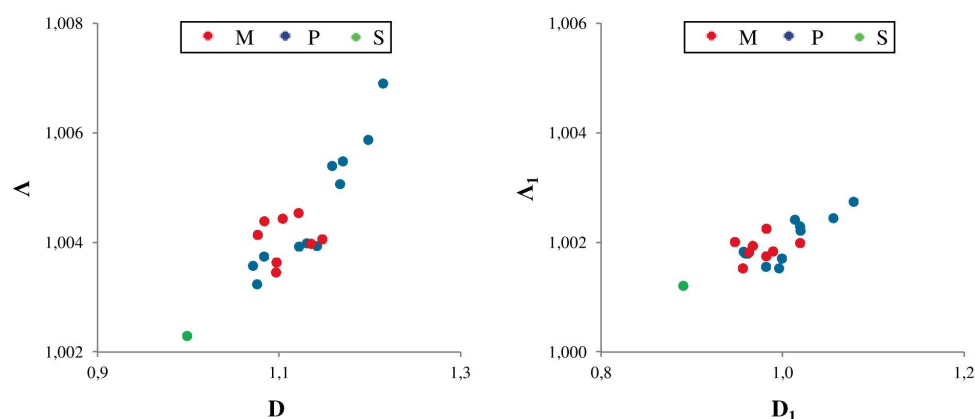


Fig. 8. Plots of the fractal dimension ( $D$  and  $D_1$ ) vs. the lacunarity ( $\Lambda$  and  $\Lambda_1$ ) for the two studied scaling laws, drainage system on the left and first-order streams on the right, for the plutonic (P), metamorphic (M), and sedimentary (S) lithologic units.

Other scaling parameters considered in this work are the reference density parameter and lacunarity. The reference density appears to be very close for the two features—drainage network and lithology—evaluated from different sources of data. Because the scaling dimensions are different, the densities diverge as the size of the boxes grows. The notion of lacunarity, which was introduced by Mandelbrot (1983), makes it possible to distinguish between geometric objects having similar fractal dimensions. Our results show that  $\Lambda$  increases as  $D$  increases in our study case. This positive relationship suggests that both parameters could be controlled by the same environmental factors and contradicts the assumption that  $\Lambda$  and  $D$  are inversely related.

## Conclusions

The lithologic control on the geometry of drainage systems was investigated using techniques of fractal analysis within 20 delimited areas with homogeneous bedrock lithology. The tectonic stability and the low climatic variability of the region allowed effective study of the lithologic control on the drainage network.

The very simple tool of monofractal analysis used, the box-counting dimension, has allowed identification of the scaling properties of the drainage system of irregular lithologic units and also the scaling properties of the sets of first-order streams, which geometrically represent scattered sets of lines.

The relationship between the fractal dimensions of both hydrologic features shows a different behavior between the metamorphic and the plutonic units that was not appreciable from the results of the traditional quantitative measures of fluvial geomorphology.

The lacunarity and the reference density parameter complement the fractal dimension to describe patterns of spatial dispersion in hydrologic features such as the sets of first-order streams.

This work indicates that using the available information in soil survey databases may be very advantageous in interdisciplinary studies.

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